NTU Journal of Pure Sceinces (2023) 2 (2) : 21-29
DOI: https://doi.org/10.56286/ntujps.v2i2

IRAQI Academic Scientific Journals

JPS NTU

ISSN: 2789-1089 EISSN: 2789-1097
NTU Journal of Pure Sciences
Available online at: https://journals.ntu.edu.iq/index.php/NTU-JPS/index

A Bayesian Approach to Confirmatory Factor Analysis with Non-normal Variables

Thanoon Y. Thanoon[1], Ahmad Najim Sheet Al-shallawi[2], Elham A. Hussain[3], Zarina Mohd Khalid[4]
1., 2., 3. Department of Statistics and Informatics Techniques, Administrative Technical College, Northern Technical University, Mosul, Iraq, thanoon.younis@ntu.edu.iq, a.n.s.al-shallawi@ntu.edu.iq, dr.elham@ntu.edu.iq, 4. Department of Mathematical Sciences, Faculty of science, University Technology Malaysia, Johor, Malaysia, zarinamkhalid@utm.my

Article Informations

Received: 26-04- 2023,
Accepted: 23-06-2023,
Published online: 30-06-2023

Corresponding author:
Name: Thanoon Y. Thanoon
Affiliation : Northern Technical University
Email: thanoon.younis@ntu.edu.iq

Key Words:
Confirmatory Factor Analysis,
Bayesian Approach,
Gibbs Sampling,
Ordinal Data.

ABSTRACT

This study aims to estimate the parameters of confirmatory factor analysis with non-normal variables using a Bayesian approach. In this study, the estimation method is (Markov chain Monte Carlo) "MCMC" simulation, and the ordinal variables have defined cut points (Gibbs sampling). To handle non-normal outcomes, censoring techniques with distinct cut-points are used.
Additional methods are interpreted, including the (Bayesian estimator), (standard deviations) "SD", and (highest posterior density) "HPD". (Quality of life) "QOL" data and observations from the OpenBUGS program are used to explain the established process.

Introduction

Confirmatory Factor Analysis (CFA) is a statistical test that establishes the statistical significance of a proposed factor composition by identifying the number of factors that will be present within a set of variables as well as the proportion of each factor to each variable. In order to analyze structural equation modeling, confirmatory factor analysis is a necessary step.

Confirmatory Factor Analysis (CFA), a type of SEM analysis, was performed after EFA to establish the "construct validity" of the factors and as an a priori stage for SEM analysis.

Both calculated variables are related to each latent variable in exploratory factor analysis. Researchers can categorize the number of selected variables in the data and which measurable variable is linked to which latent variable using confirmatory factor analysis (CFA) (1).

(CFA) (2) is a real system for viewing a series of relevant data in order to assess the interrelationships between manifest and latent variables.

Several scientists have proposed models based on confirmatory factor analysis in the past years. Any of these papers were suggested by Zhang et al., (3); Taylor, (4); Yu et. al, (5); Zarei et. al, (6); Karatza, (1); Shigemasu et al., (7); Hoofs, et al., (8).

Confirmatory factor analysis is an effective technique in many applications, including cross-cultural studies. The demand for unobtrusive models and related real strategies for addressing complex research issues in various fields has prompted the rapid development of confirmatory factor analysis models.

The concept of a Bayesian technique, in which the censoring distribution and latent variables are considered as fictitious missing data, is produced by the Gibbs sampling algorithm (9) Conjugate priors are employed for the structural parameters, and the cut points are determined by non-informative priors.

In this paper, a method for analyzing confirmatory factor analysis models with non-normal data is given using a Bayesian approach. For model selection, the Deviance Information Criterion (DIC) is used (10).

The primary challenge is to address the issue of non-normal data (ordinal data) in the confirmatory factor analysis. Bayesian analysis is used to estimate the parameters, regard the censoring distribution as missing data, and combine it with the data in the posterior analysis.

The following is the structure of the paper. Part 2 introduces the confirmatory factor analysis model. In Part 3, Bayesian estimation is used to implement confirmatory factor analysis. Part 4 employs DIC to address Bayesian model selection. A actual data example is presented in Part 5. In Part 6, The research results and comments are covered, and the conclusions are provided in Part 7.

Confirmatory Factor Analysis Model

The exploratory factor analysis (EFA) paradigm naturally leads to the confirmatory factor analysis (CFA) model. For illustration, the following CFA is regarded as (2).

$$\boldsymbol{x} = \boldsymbol{\Lambda}\xi + \varepsilon, \qquad (1)$$

where $\boldsymbol{x}_i$ $(p \times 1)$ is introduced as a random vector of observed variables, $\boldsymbol{\Lambda}\,(p \times q)$ is a factor loading matrix, $\boldsymbol{\xi}$ $(q \times 1)$ is a latent factors vector, $\boldsymbol{\varepsilon}(p \times 1)$ is called a random vector of residuals. So, $\boldsymbol{\xi}$ is then distributed as $N[\boldsymbol{0}, I]$. Further, $\boldsymbol{\varepsilon}$ is distributed as $N[\boldsymbol{0}, \boldsymbol{\Psi}_\varepsilon]$, where Ψ_ε with diagonal elements $\psi_{\varepsilon 1}, \dots, \psi_{\varepsilon p}$ is a diagonal matrix. It has also been determined that, in this case, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\xi}$ are both independent as well.

The confirmatory factor analysis proposed here can be applied to a wide range of circumstances. Furthermore, when it comes to confirmatory factor analysis, it's important to pay attention to how the mean vector, x, relates to μ.

To address the issue of non-normal data, it should be assumed that is a sub-vector of the unobservable continuous data. The observed ordinal vector $\boldsymbol{z}_i$ reflects this information. An ordinal variable can be introduced in accordance with its unobserved continuous random variable $\boldsymbol{y}_m$ at the most fundamental level.

Such that it is also true that $\{-\infty = \alpha_{m,1} < \alpha_{m,2} < ... < \alpha_{m,b_m} < \alpha_{m,b_m+1} = \infty\}$ is the set of cut point's specification which define the given classes, and b_m for which b_m represents the number of selected cut points regarding the ordinal variable $\boldsymbol{z}_m$.

However, it must be understood that the number of cut-points is similar for every ordinal variable.

Bayesian Analysis of Confirmatory Factor Analysis Model

In this part, we'll look at confirmatory factor analysis and how to estimate the parameters using the Bayesian approach. Enable, on the other hand, $\boldsymbol{\theta}$ a vector representing the unidentified parameter in the aforementioned model, as well as $\boldsymbol{\alpha}$ serving as a vector representing the cut points for the ordinal variables.

To be more precise, we permit $\boldsymbol{\theta}$ to be a vector holding the entire collection of unique unknown parameters. As a result, the Gibbs sampler is used to construct the Bayesian estimates of θ and $\boldsymbol{\alpha}$.

In addition, consider $\boldsymbol{Z} = (\boldsymbol{z}_1, ..., \boldsymbol{z}_N)$ the observed ordinal data and $\boldsymbol{Y} = (\boldsymbol{y}_1, ..., \boldsymbol{y}_N)$ the latent continuous measurement with regard to Z.

Add Y to the observed data in the posterior analysis after that. The problem will be simpler to solve once Y has been defined because all the data are taken into consideration and are treated continuous. Allow $\boldsymbol{\Omega} = (\boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_N)$ the matrix of latent variables to be as well.

As a result, by enhancing the data, problems related to the model's more intricate components can be resolved. Posterior analysis Z, which stands for the collection of observed data, can be added to by $(\boldsymbol{Y}, \boldsymbol{\Omega})$. We will also show the joint posterior distribution $[\boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{Y}, \boldsymbol{\Omega} | \boldsymbol{Z}]$. To produce a set of observations from the relevant joint posterior distribution, the Gibbs sampling algorithm can be used.

The sample mean and variance matrices can be used to calculate the Bayesian estimate $\boldsymbol{\theta}$ for and the standard error estimates, respectively.

$$\theta = N^{-1} \sum_{t=1}^{N} \theta^{(t)}, \ \mathrm{Var}(\theta/Z) = (N-1)^{-1} \sum_{t=1}^{N} (\theta^{(t)} - \theta)(\theta^{(t)} - \theta)^{/} \quad (2)$$

The following well-known conjugate prior distributions are used:

$$p(\lambda_k) \sim N[\lambda_{0k}, \boldsymbol{H}_{0k}], \ p(\lambda_{\xi k} | \psi_{\delta k}) \sim N[\lambda_{0\xi k}, \psi_{\delta k} \boldsymbol{H}_{0\xi k}],$$

$$p(\boldsymbol{\Phi}^{-1}) \sim W_q[\boldsymbol{R}_0, \rho_0], \quad p(\psi_{\delta k}^{-1}) \sim Gamma[\alpha_{0k}, \beta_{0k}] \quad (3)$$

Given the definition that $\psi_{\delta k}$, is the kth diagonal element of $\boldsymbol{\Psi}_\delta, \lambda_k'$ and $\lambda_{\xi k}'$ are the kth rows of $\boldsymbol{\Lambda}$ and $\boldsymbol{\Lambda}_\xi$, respectively. $\boldsymbol{H}_{0\mu} = diag(\sigma_{01}^2, ..., \sigma_{0p}^2)$, and $\lambda_{0k}, \lambda_{0\xi k}, \alpha_{0k}, \ \beta_{0k}, \rho_0, \sigma_{0k}, \boldsymbol{H}_0, \boldsymbol{H}_{0\xi k}$, and $\boldsymbol{R}_0$ are assumed to be known, however, prior information is obtained via analysis of past data, theoretical consideration, and causal observance.

As built up incited by Kass and Raftery (11), expected prior knowledge, as it is connected to current models, are commonly chosen only for comfort when there is no sufficiently precise gathered prior data. This should be possible on the grounds that the impact these presumptions have on Bayesian estimations stays little, notwithstanding when a huge example measure is utilized. The outcomes are useful when attempting to utilize PC displaying with the Gibbs sampler (9) because it can simulate $\boldsymbol{\alpha}, \boldsymbol{\theta}$ and $\boldsymbol{\Omega}$, all from the conditional distribution.

Notwithstanding, because of the presence of ordinal data for this situation, the related conditional distributions can be made excessively unpredictable, making it impossible to easily derive or simulating data from them.

To get on the observation from the posterior distribution, the simulation process will begin with the starting values and then simulate the first observation $(\boldsymbol{\alpha}^{(1)}, \boldsymbol{\theta}^{(1)}, \boldsymbol{\Omega}^{(1)}, \boldsymbol{Y}^{(1)})$ and finally simulate the mth repetition of these observations.

The series, after the mth repetition, will give us $(\boldsymbol{\alpha}^{(m+1)}, \boldsymbol{\theta}^{(m+1)}, \boldsymbol{\Omega}^{(m+1)}, \boldsymbol{Y}^{(m+1)})$. However, the joint distribution $(\boldsymbol{\alpha}^{(m)}, \boldsymbol{\theta}^{(m)}, \boldsymbol{\Omega}^{(m)}, \boldsymbol{Y}^{(m)})$ can be proven to move toward the joint posterior distribution $[\boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\Omega}, \boldsymbol{Y} \,|\boldsymbol{Z}]$ (Geyer, 1992) .(12)

Bayesian Model Selection

In this paper, DIC is used provided by OpenBUGS. It is equal to:

$$DIC = \bar{D} + p_D = D(\bar{\theta}) + 2p_D \quad (4)$$

Where

$\bar{D}$ is (the posterior mean of the deviance),

p_D is (the effective number of parameters),

and $D(\bar{\theta})$ is the point estimate of the deviance at the mean of the estimated parameters θ.

The deviance is defined as

$$D(\theta) = -2\log f\ (y\ /\theta) \quad (5)$$

While

$$\bar{D} = -\frac{2}{N}\sum_{t=B+1}^{N} \log f\ (y\ /\theta^{(t)}) \quad (6)$$

The model that can best predict a replicate dataset with a structure that differs from what is currently observed has the minimal DIC, according to this theory (1).

The OpenBUGS software generates the DIC values for the confirmatory factor analysis with real data.

Real Data Example

The analysis of the data gathered for the aim of this study will be thoroughly covered in this part. The data is incredibly helpful for clinical work, social insurance planning and evaluation, and medical research. It measures the quality of life. It is commonly acknowledged that quality of life (QOL) is a multifaceted notion best assessed by different underlying constructs, including physical ability, wellbeing status, mental status, and social relations (13).

A QOL instrument typically has three to five categories and items that are estimated on an ordinal scale. The items' discrete ordinal character also attracts much attention in QOL research (14); (15).

This instrument WHOQOL-100 (13) was built up to assess four latent factors.

Table 1. Description of the data

" (Q1 to Q7) are planned to address physical health"
"(Q8 to Q13) are proposed to address mental health"
" (Q14, Q15, Q16) that take after are for social connections"
"(Q17 to Q24) are expected to address condition, giving a sum of 24 items"

A five-point scale is used to rate the majority of the items (1 = "not at all/very dissatisfied," 2 = "a little/dissatisfied," 3 = "moderate/neither," 4 = "very much/satisfied," and 5 = "extremely/very satisfied").

To outline the Bayesian techniques, we investigate a real data set with sample size n = 338.

Regarding these ordinal data as originating from continuous normal distribution is not right.

A real data example with 24 manifest variables are related to four basic latent variables $(\xi_{i1}, \xi_{i2}, \xi_{i3}, \xi_{i4})$ from CFAMs defined in Equation (3) is discussed here. To illustrate the confirmatory factor analysis Bayesian method with ordinal variables, using an actual data set that has random vector $\boldsymbol{z}_i = (\boldsymbol{z}_{i1}, \boldsymbol{z}_{i2}, ..., \boldsymbol{z}_{i24})'$, consider $\boldsymbol{y}_i = (\boldsymbol{y}_{i1}, \boldsymbol{y}_{i2}, ..., \boldsymbol{y}_{i24})'$ the random vector of latent continuous, which corresponds to the ordinal variables $\boldsymbol{z}_{i1}, \boldsymbol{z}_{i2}, ..., \boldsymbol{z}_{i24}$ where $\boldsymbol{z}_i$, $\mathrm{i}=1,...,\mathrm{n}$. The ordinal variables are connected to (4) latent variables, $\boldsymbol{w}_i = (\xi_{i1}, \xi_{i2}, \xi_{i3}, \xi_{i4})$, $\boldsymbol{\varepsilon}_i = (\boldsymbol{\varepsilon}_{i1}, \boldsymbol{\varepsilon}_{i2}, ..., \boldsymbol{\varepsilon}_{i24})$ with the parameter values in $\boldsymbol{\Lambda} = (\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, ..., \boldsymbol{\lambda}_{20})'$.

The confirmatory factor analysis model evaluates the associations of the latent variables in $w_i = (\xi_{i1}, \xi_{i2}, \xi_{i1}, \xi_{i2})$.

The following were the hyper-parameter values' previous inputs:

1. Prior I: The elements in λ_{0k} and $\lambda_{0\xi k}$ Equations (3) and are series equal to the following possible values, with initial values of 1.

"$\boldsymbol{R}_0^{-1} = 8\boldsymbol{\Phi}$, $\boldsymbol{H}_{0u}, \boldsymbol{H}_{0k}$ and $\boldsymbol{H}_{0\xi k}$ are used to be 0.25 times the identity matrices; $\alpha_{0k} = 10$, $\beta_{0k} = 8$, $\rho_0 = 30$".

2- Prior II: The following optional values, with initial values equal to 0.5, are placed between the elements in λ_{0k} and $\lambda_{0\xi k}$ in Equation (3);

"$\boldsymbol{R}_0^{-1} = 8\boldsymbol{\Phi}$, $\boldsymbol{H}_{0u}, \boldsymbol{H}_{0k}$ and $\boldsymbol{H}_{0\xi k}$ are used to be 0.25 times the identity matrices; $\alpha_{0k} = 10$, $\beta_{0k} = 8$, $\rho_0 = 30$".

OpenBUGS (16) used the quality of life data set with (n=338) to implement Bayesian estimates in the confirmatory factor analysis model. When comparing the Bayesian analyses of confirmatory factor analysis with the data, the MCMC process for data analysis required more iterations to converge. In the confirmatory factor analysis model, the parameters were estimated using a Bayesian technique from T=10000 iterations for the censored distribution. We will use OpenBUGS to analyze the current data set in order to explain it.

Results and Discussion

This section's goal is to discuss data results in order to show how well the parameter estimates and model choice work empirically.

For analyzing confirmatory factor analysis models for ordinal data, the Bayesian technique is introduced. Utilizing recently created robust tools and the open-source statistical program OpenBUGS, the Bayesian analysis of the unknown parameters and the model selection (DIC) are obtained. As a result, using our suggested method with actual data is convenient. Utilizing Bayesian confirmatory factor analysis models with ordinal variables is the goal of this analysis. The examination of ordinal data has some restrictions in confirmatory factor analysis models. For analyzing confirmatory factor analysis models for ordinal data, the Bayesian technique is introduced. Utilizing recently created robust tools and the open-source statistical program OpenBUGS, the Bayesian analysis of the unknown parameters and the model selection (DIC) are obtained. As a result, using our suggested method with actual data is convenient. Utilizing Bayesian confirmatory factor analysis models with ordinal variables is the goal of this analysis. The examination of ordinal data has some restrictions in confirmatory factor analysis models.

To begin with, despite the presence of discrete data, most data in the social and behavioral sciences are ordinal data. Finding a different approach to manage the issue of ordinal variables is important because ordinal data analysis obviously violates the main hypothesis of CFA, which is that the data come from a normal distribution.

As a result, systematically treating ordinal factors as ordinary can lead to erroneous conclusions (17). Treating these data as findings that come from a censoring distribution with specific cut points is a better method for evaluating them.

Figure (2) below explains the path diagram of CFA model and the relationships between latent variables and observed variables.

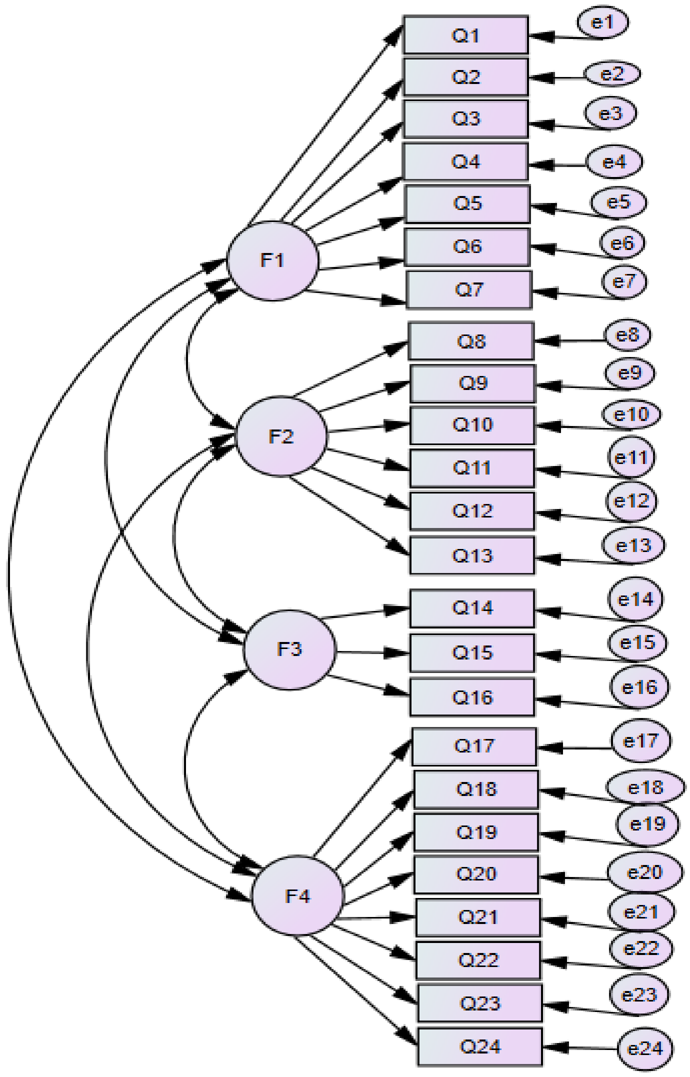

Figure 1. Path Diagram

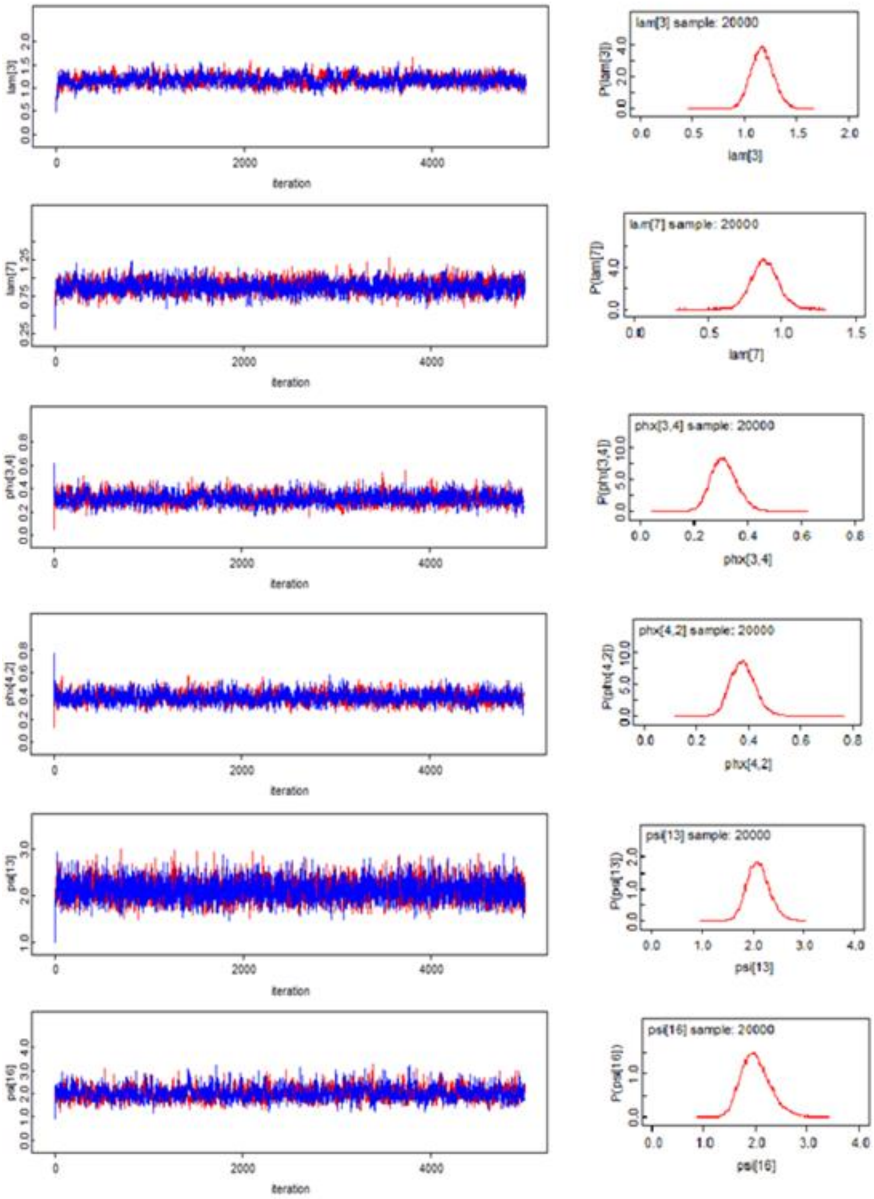

Figure 2. Time Series Plot and Posterior Density Plot of observation corresponding to (a) λ_3 ; (b) λ_7 ; (c) Φ_{34}; (d) Φ_{42} ; (e) Ψ_{13}; (f) Ψ_{16} ; and for CFA with Ordinal Variables

Table 2. Bayesian Estimation of Confirmatory Factor Analysis Model with Ordinal Variables

(Par)	(Est)	(SD)	(HPD)	(Par)	(Est)	(SD)	(HPD)
λ_1	1.046	0.113	[0.835, 1.279]	ϕ_{33}	0.575	0.093	[0.408, 0.775]
λ_2	1.22	0.100	[1.035, 1.422]	ϕ_{34}	0.312	0.050	[0.221, 0.416]
λ_3	1.167	0.107	[0.969, 1.381]	ϕ_{44}	0.489	0.064	[0.379, 0.623]
λ_4	0.954	0.102	[0.759, 1.164]	Ψ_1	0.866	0.089	[0.705, 1.054]
λ_5	1.176	0.084	[1.021, 1.349]	Ψ_2	0.593	0.066	[0.472, 0.731]
λ_6	1.292	0.101	[1.095, 1.492]	Ψ_3	1.294	0.144	[1.031, 1.591]
λ_7	0.883	0.086	[0.717, 1.056]	Ψ_4	0.901	0.105	[0.711, 1.119]
λ_8	0.693	0.073	[0.555, 0.842]	Ψ_5	0.724	0.072	[0.593, 0.869]
λ_9	0.821	0.091	[0.650, 1.008]	Ψ_6	3.257	0.401	[2.536, 4.088]
λ_{10}	1.058	0.082	[0.899, 1.223]	Ψ_7	1.428	0.159	[1.142, 1.764]
λ_{11}	0.849	0.074	[0.705, 0.997]	Ψ_8	2.237	0.246	[1.785, 2.747]
λ_{12}	0.276	0.131	[0.025, 0.537]	Ψ_9	1.370	0.139	[1.115, 1.656]

λ_{13}	0.978	0.109	[0.778,	1.209]	Ψ_{10}	1.824	0.173	[1.504, 2.183]
λ_{14}	0.832	0.087	[0.666,	1.006]	Ψ_{11}	1.195	0.122	[0.970, 1.454]
λ_{15}	1.044	0.113	[0.829,	1.274]	Ψ_{12}	2.265	0.255	[1.804, 2.800]
λ_{16}	0.774	0.086	[0.608,	0.947]	Ψ_{13}	2.107	0.218	[1.714, 2.562]
λ_{17}	1.059	0.117	[0.835,	1.301]	Ψ_{14}	1.988	0.286	[1.503, 2.616]
λ_{18}	0.744	0.101	[0.550,	0.946]	Ψ_{15}	0.528	0.049	[0.437, 0.628]
λ_{19}	0.636	0.080	[0.484,	0.799]	Ψ_{16}	2.004	0.283	[1.509, 2.618]
λ_{20}	0.682	0.092	[0.507,	0.869]	Ψ_{17}	2.374	0.255	[1.910, 2.908]
ϕ_{11}	0.752	0.110	[0.572,	0.978]	Ψ_{18}	1.589	0.156	[1.306, 1.920]
ϕ_{12}	0.453	0.059	[0.348,	0.575]	Ψ_{19}	0.973	0.098	[0.795, 1.181]
ϕ_{13}	0.224	0.053	[0.127,	0.332]	Ψ_{20}	1.649	0.164	[1.351, 1.989]
ϕ_{14}	0.397	0.054	[0.302,	0.509]	Ψ_{21}	0.867	0.085	[0.713, 1.042]
ϕ_{22}	0.562	0.070	[0.436,	0.712]	Ψ_{22}	1.153	0.126	[0.916, 1.410]
ϕ_{23}	0.322	0.052	[0.228,	0.433]	Ψ_{23}	1.812	0.175	[1.488, 2.180]
ϕ_{24}	0.383	0.048	[0.297,	0.484]	Ψ_{24}	1.345	0.139	[1.090, 1.630]

Table 3. Confirmatory Factor Analysis Model with Ordinal Variables and Goodness of Fit Statistics(DIC)

	Dbar	Dhat	DIC	pD
Total	18290.0	17400.0	19190.0	897.7

This article reports results of Bayesian confirmatory factor analysis performed to QOL data.

Table 2 shows the results for ordinal variables with censoring distributions and cut point specifications (2). We found that the SD values for all parameters are small.

The parameter with the highest posterior density (HPD) was chosen. When using a censored distribution, we discovered that the HPD intervals are useful for ordinal variables. The HPD is far from zero, so all parameters are important.

We used confirmatory factor analysis to conduct a second examination of the data sets in order to demonstrate the utility of DIC for the goodness of fit test. The right model was used to compare the DIC values that were obtained. The outcomes are shown in Table (3).

When employing censored distribution and ordinal data, the goodness of fit test for CFA is (19190.0).

With cut points specified, the best-fitted model with the minimum DIC value was taken into consideration (-200.000, -2.517, -1.245, -0.444, 0.848, 200.000). Therefore, DIC's performance was satisfactory.

Plots illustrating many sequences of the different parameters that were produced, with varying initial values were used to observe the Gibbs sampler's convergence, which is shown separately in Figure (2). In CFA models for censoring distribution, Bayesian evaluations were obtained from T = 10000 iterations.

Conclusions and Recommendations

As seen in this article, social and behavioral sciences frequently employ confirmatory factor analysis models. This paper can be divided into three parts. To obtain all of the predicted parameters, the first step in this analysis was to use confirmatory factor analysis models. The second argument was to use censoring distribution to solve the problem of ordinal data.

Cut points were used to carry out the proposed procedures. The development of confirmatory factor analysis models is the third stage. There are representations of latent variables in equations in confirmatory factor analysis models. Due to the complication of the suggested model, this article uses observable techniques to obtain "SD" estimates, as well as a Bayesian goodness of fit test using the (DIC) are discussed.

As we've seen, the discrete concept of ordinal data and the causal links among latent components are problems that can be solved by data augmentation using some MCMC approaches. With complete knowledge, it will be rather simple to add the hypothetically missing data to the actual observed data. This is a powerful technique that can be linked to other, more uncertain techniques.

References

[1] Alkuwaiti, A., Maruthamuthu, T., & Akgun, S. (2020). Factors associated with the quality of outpatient service: The application of factor analysis–A case study. International Journal of Healthcare Management, 13(sup1), 88-93.

[2] Geman, S. and Geman, D. (1984). Stochastic relaxation,Gibbs distribution,and the Bayesian restoration of images. IEEE Transactions on Pattern Analysis and Machine Intelligence, (6), 721-741.

[3] Geyer, C. J. (1992). Practical markov chain monte carlo. Statistical Science, 473-483.

[4] Hoofs, H., van de Schoot, R., Jansen, N. W., & Kant, I. (2018). Evaluating model fit in Bayesian confirmatory factor analysis with large samples: Simulation study introducing the BRMSEA. Educational and Psychological Measurement, 78(4), 537-568.

[5] Karatza, A. S. (2006). Bayesian Factor Analysis: Implementation on Schizotypal Personality Disorder Data (Doctoral dissertation, Athens University of Economics and Business).

[6] Kass, R. E. and Raftery, A. E. (1995). Bayes factors. Journal of the american statistical association, 90(430), 773-795.

[7] Lee, S.- Y. (2007). Structural equation modeling: A Bayesian approach (Vol. 711). John Wiley & Sons.

[8] Mair, P. (2018). Factor analysis. In Modern Psychometrics with R (pp. 17-61). Springer, Cham.

[9] Power, M., Bullingen, M., Hazper, A. and WHOQOL Group (1999) The World Health Organization WHOQOL-100: tests of the universality of quality of life in 15 different cultural groups worldwide. Health Psychology, 18, 495–505.

[10] Shigemasu, K., Kono, M., & Ueno, K. (2020). Bayesian confirmatory factor analysis of Wechsler Intelligence Scale for children data. Behaviormetrika, 47(2), 451-467.

[11] Spiegelhalter, D. J., Best, N. G., Carlin, B. P., and & van der Linde, A. (2002). Bayesian measures of model complexity and fit (with discussion). Journal of the Royal Statistical Society, Series B, 64(4), 583-639.

[12] Spiegelhalter, D., Thomas, A., Best, N. G. and Lunn, D. (2007). OpenBUGS user manual. Medical Research Council Biostatistics Unit, Cambridge.

[13] Taylor, J. M. (2019). Overview and illustration of Bayesian confirmatory factor Analysis with ordinal indicators. Practical Assessment, Research, and Evaluation, 24(1), 4.

[14] Thanoon, T. Y., Hamed, A. T., & Adnan, R. (2019). Bayesian Nonlinear Latent variable Models with Mixed Non-normal Variables and Covariates for Multi-sample Psychological Data. Pakistan Journal of Statistics and Operation Research, 627-645.

[15] Yu, H., Campbell, M. T., Zhang, Q., Walia, H., & Morota, G. (2019). Genomic bayesian confirmatory factor analysis and bayesian network to characterize a wide spectrum of rice phenotypes. G3: Genes, Genomes, Genetics, 9(6), 1975-1986.

[16] Zarei, E., Karimi, A., Habibi, E., Barkhordari, A., & Reniers, G. (2021). Dynamic occupational accidents modeling using dynamic hybrid Bayesian confirmatory factor analysis: An in-depth psychometrics study. Safety science, 136, 105146.

[17] Zhang, J., Templin, J., & Mintz, C. E. (2021). A Model Comparison Approach to Posterior Predictive Model Checks in Bayesian Confirmatory Factor Analysis.